Designing Effective Observing Network for Data Assimilation based on Sparse Sensor Placement Method

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Summary

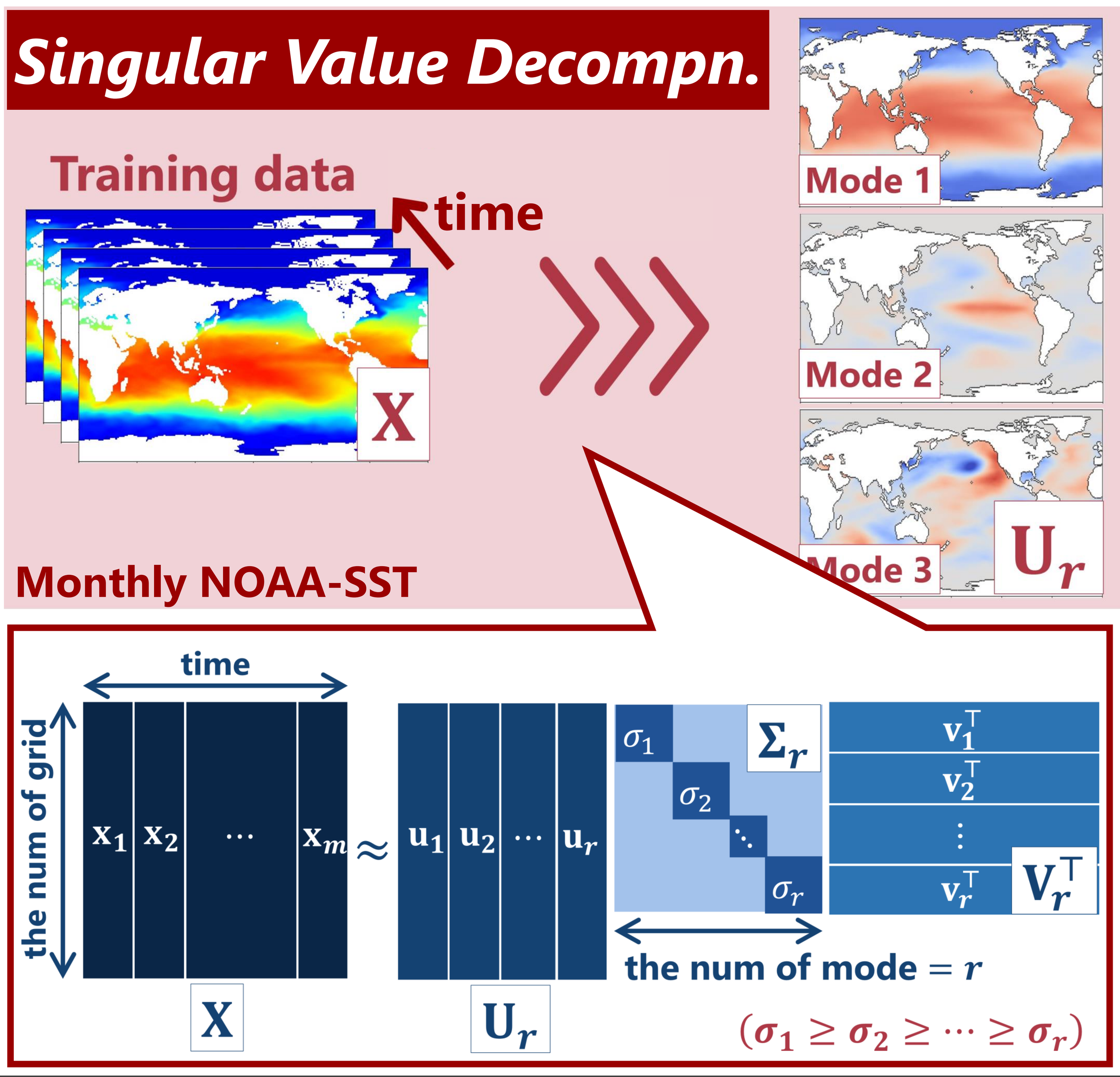
- We aim to develop a sensor selection method suitable for data assimilation (DA).
- Sparse Sensor Placement (SSP) is a sensor selection method developed in information engineering.
- The most suitable SSP for DA is the conventional SSP method, BDG, considering singular value and regularization term.
- BDG doesn't need tunings of the number of modes.

Bayesian-based Determinant Greedy; BDG

Sparse Sensor Placement (SSP)

SSP is the sensor selection method to minimize the estimation error of the low-dimensional state vector a.

Singular Value Decompn.



Dimension reduction $\mathbf{x} \approx \mathbf{U}_r\mathbf{a} \quad \mathbf{a} \in \mathbb{R}^r$

Obs. vector $\mathbf{y} = \mathbf{H}\mathbf{x} \approx \mathbf{H}\mathbf{U}_r\mathbf{a} = \mathbf{C}\mathbf{a}$

3 different methods for SSP

Minimization of the estimation error of a

Method	Estimation of vector a	Conditions of the opt. obs.
HMODE (Saito *et al.* 2021)	$\hat{\mathbf{a}} = \begin{cases} \mathbf{C}^\top(\mathbf{C}\mathbf{C}^\top)^{-1}\mathbf{y} & (p \leq r) \\ (\mathbf{C}^\top\mathbf{C})^{-1}\mathbf{C}^\top\mathbf{y} & (p > r) \end{cases}$	$\begin{cases} \mathrm{argmax}\det \mathbf{C}\mathbf{C}^\top & (p \leq r) \\ \mathrm{argmax}\det \mathbf{C}^\top\mathbf{C} & (p > r) \end{cases}$
BDG (Yamada *et al.* 2021)	$\hat{\mathbf{a}} = [\mathbf{\Sigma}_r^{-2} + \mathbf{C}^\top\mathbf{R}^{-1}\mathbf{C}]^{-1}\mathbf{C}^\top\mathbf{R}^{-1}\mathbf{y}$	$\mathrm{argmax}\det[\mathbf{\Sigma}_r^{-2} + \mathbf{C}^\top\mathbf{R}^{-1}\mathbf{C}]$

HMODE + Singular Value $\mathbf{\Sigma}_r$

Method		Conditions of the opt. obs.
HSQRT (Saito *et al.* 2022)	$\mathbf{D} = \mathbf{C}\mathbf{\Sigma}_r = \mathbf{H}\mathbf{U}_r\mathbf{\Sigma}_r$	$\begin{cases} \mathrm{argmax}\det \mathbf{D}\mathbf{D}^\top & (p \leq r) \\ \mathrm{argmax}\det \mathbf{D}^\top\mathbf{D} & (p > r) \end{cases}$

p: the num. of obs. r: the num. of modes

	Singular value	*Regularization* R
HMODE		
HSQRT	✓	
BDG	✓	✓

SSP for data assimilation and Experiment

Analysis error covariance matrix

$$\mathbf{P}^a = \mathbf{Z}^b\left[\mathbf{I} + (\mathbf{H}\mathbf{Z}^b)^\top\mathbf{R}^{-1}(\mathbf{H}\mathbf{Z}^b)\right]^{-1}(\mathbf{Z}^b)^\top$$

$$\mathbf{Z}^b \approx \mathbf{U}_r\mathbf{\Sigma}_r\mathbf{V}_r^\top$$

Analysis error covariance matrix

$$\mathbf{P}^a \approx \mathbf{U}_r[\mathbf{\Sigma}_r^{-2} + (\mathbf{H}\mathbf{U}_r)^\top\mathbf{R}^{-1}(\mathbf{H}\mathbf{U}_r)]^{-1}(\mathbf{U}_r)^\top$$

Minimization of the analysis error

Condition of the opt. obs.

$$\mathrm{argmax}\det[\mathbf{\Sigma}_r^{-2} + (\mathbf{H}\mathbf{U}_r)^\top\mathbf{R}^{-1}(\mathbf{H}\mathbf{U}_r)]$$

This condition is the same as BDG.
→ BDG is the SSP method for DA, mathematically.

Experimental settings

Data: Monthly NOAA-SST

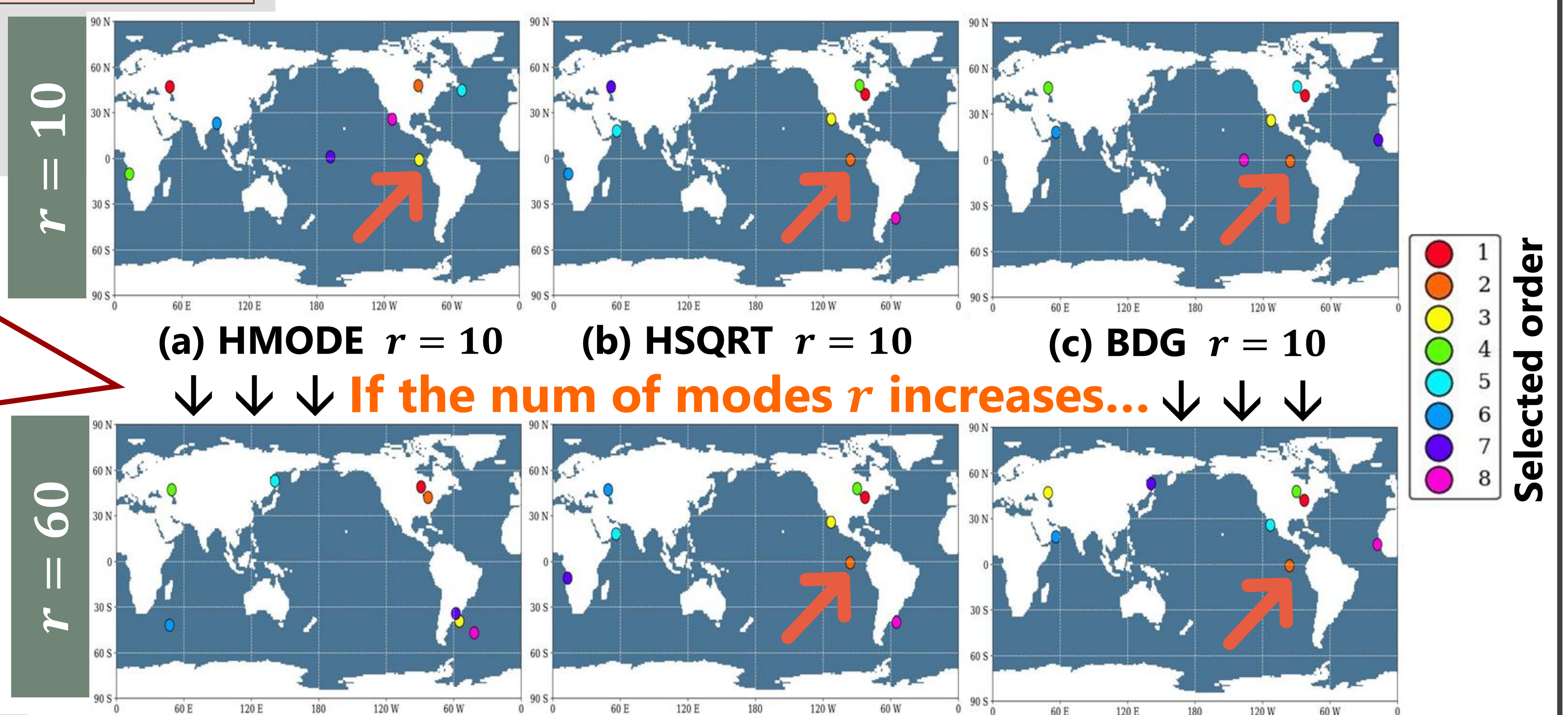
Training: 1995-1999 Validation: 2005-2014

Experimental procedure

1. To select obs. points using SSP
2. To generate obs. data by adding noise $N(0, 1)$ to validation data
3. To estimate SST fields from obs. using data assimilation
   $\bar{\mathbf{x}}^b$: temporal average of the training data $\mathbf{X}$
   $\mathbf{Z}^b$: deviation of $\mathbf{X}$ from the temporal average $\bar{\mathbf{x}}^b$

Sensor locations determined with SSP

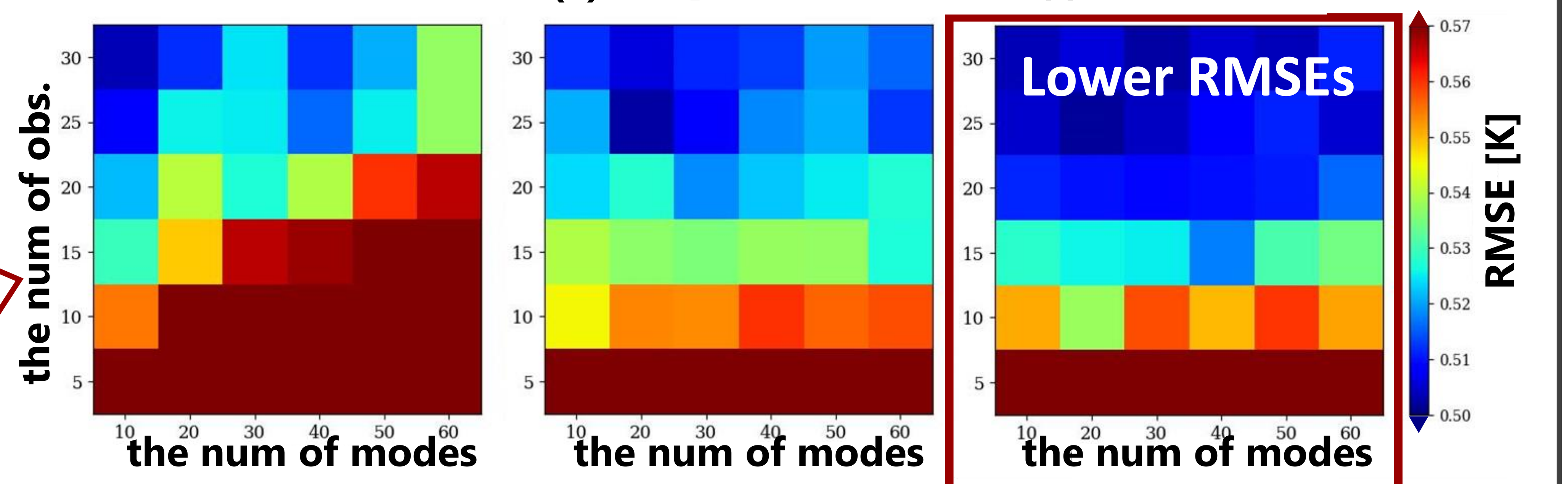
- When $r = 10$, sensors are placed in offshore Peru to represent the ENSO mode.
- When $r = 60$
  HMODE: No sensors represent ENSO.
  HSQRT, BDG: Some sensors represent ENSO.



(a) HMODE $r = 10$ (b) HSQRT $r = 10$ (c) BDG $r = 10$
(d) HMODE $r = 60$ (e) HSQRT $r = 60$ (f) BDG $r = 60$

- **HMODE:** RMSE **increases** with r.
- **HSQRT, BDG:** RMSE is **insensitive** to r.
- **Singular values represent the amplitude of each mode.**

→ **We don't need the tuning of the num. of modes with HSQRT or BDG .**



(a) HMODE (b) HSQRT (c) BDG

r: the num. of modes